Assegno di ricerca

"Stochastic processes in non-homogeneous media as models of anomalous dynamics"

Scientific description

This postdoctoral position *(assegno di ricerca)* stems from the joint research project *"Models of anomalous dynamics in disordered and random media"*, a convention between the Department of Physics and Astronomy of University of Bologna, the Department of Science and Advanced Technology of University of Insubria and the Department of Mathematics and Applications of University of Milano-Bicocca. The successful candidate will hold a PhD in Physics, Mathematics, or other denominations providing a strong background in mathematical physics.

The topic of investigation will be stochastic processes exhibiting anomalous dynamics, that is, processes whose probabilistic properties differ, in one or more aspects, from those of the brownian motion. The main interest will be on superdiffusive models coming from Physics, or the sciences in general, especially those whose superdiffusive properties are due to non-homogeneous, disordered and random media (as they naturally arise from applications). Among these, a model of sure interest is the *Lévy-Lorentz gas* introduced by Barkai, Fleurov and Klafter in 2000, a mathematically rigorous description of which is still largely unknown. One of the goals of the project is also to extend this model to higher dimension, a non-obvious task in itself, and to consider similar models better suited to real-world applications.

The methodology will be that of exact results, with a preference for mathematical theorems, though computer simulations will all but certainly be a necessary tool, both for intuiting the correct equations and as valuable sources of information in themselves. A non-exhaustive list of questions of interest is as follows.

Scaling and limit laws. The most natural question concerning models of random walks in Lévy media is precisely to check whether they exhibit anomalous diffusion or not, that is, to study the asymptotic scalings of the process and of its moments. We would also like to achieve limit theorems for such scalings (functional limit theorems, when possible). Even for the prominent case of the Lévy-Lorentz gas proper, the current knowledge is limited. Numerical simulations and approximate arguments have been given to conjecture the scalings of the distribution and of the moments for a range of parameters but other ranges are not covered. Moreover, very few of these conjectures have been rigorously proven. One of our main goals is precisely to fill these gaps. Much less is known about other models in Lévy media. A puzzling problem in this area is even how to produce suitable generalizations of the Lévy-Lorentz gas in dimension 2 and higher. Some models have been proposed but virtually no work has been done on them.

First-passage events. The study of first-passage events for stochastic processes represents a fundamental problem in fluctuation theory, with a plethora of applications. An important question in this area is how a process can be optimized. One very fruitful strategy, as shown in recent years, is stochastic resetting, which consists in restarting the process after random time intervals until the first-passage event occurs. In particular, stochastic processes giving rise to anomalous diffusion, if unconstrained, proved to generate a particular rich scenario under resetting. While the temporal aspect of first-passage events has been a prominent focus in the existing literature, the spatial properties of the problem have been partially overlooked. Stochastic resetting represents a viable strategy for controlling the first-passage position too. The main objective of this part of the project is to examine the attributes of a specific resetting protocol that make the optimization of the first-passage position possible.

Statistics of extrema. Connected to the above theme, a momentous topic is the study of extrema of superdiffusive processes. Many of the questions that are natural to ask fall under the umbrella of Extreme Value Theory (EVT). The processes of interest for this project are known or expected to have rather peculiar features in terms of the statistics of extrema, cf., e.g., the so-called *Big Jump Principle*. Another topic of interest, especially in the recent mathematical literature on the subject, is that of large deviations. Large deviation theory has many connections with EVT, some clear, some less explored. One goal is to improve the Large Deviation Principle for sums of not necessarily independent variables in the domain of attraction of stable distributions. One topic that in not usually investigated within the realm of EVT, that has to do with leaking (a.k.a. open) systems, is that of large deviations of extrema. We are especially interested in a quantitative control of the *survival rate* as a function of the parameters defining a given extreme event.

Activity plan

The detailed activity plan of the postdoctoral position will largely depend on the specific problem(s) the postdoctoral fellow will work on. In any event, on top of regular meetings with the supervisor at the Department of Physics and Astronomy the University of Bologna, the fellow will interact with the other project members at University of Insubria and University of Milano-Bicocca and will travel to national/international conferences to stay updated in the research field and to disseminate their results. Scientific visits to experts in the topics of interest will also be encouraged.